Rules of upscaling for rock physics transforms: Composites of randomly and independently drawn elements

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\textbf{ABSTRACT}

The tight trends (transforms) between two rock physical properties obtained in the physical or digital laboratory on cm- or mm-sized samples may hold for a composite constructed of these samples. One requirement is that all the elements of the composite obey the same trend. Another requirement is that the composite is spatially uncorrelated and, hence, approximately isotropic. The final requirement is that the underlying elemental transforms are approximately linear. The methods we use to address this exportability of trends for the dynamic elastic moduli (or the elastic-wave impedance) versus porosity, permeability versus porosity, and electrical resistivity versus porosity are, respectively, the theoretical elastic bounding, numerical reservoir-scale fluid flow, and electrical current simulations. A practical implication is that if an elastic property of a large volume is determined from remote sensing, be it seismic or cross-well data, its average porosity can be estimated using the transforms established at a much smaller scale. Then, this average porosity can be translated into the hydraulic and electrical properties once again, using the elemental transforms established in the physical or digital laboratory.

\textbf{INTRODUCTION}

“Upscaling” is one of the several geophysical concepts whose exact meaning varies with the task under examination and, often, with the expert who addresses this task. The traditional meaning of upscaling is often associated with the Backus (1962) elastic average, where the effective elastic moduli of a layered sequence in the direction normal to the layers is the harmonic average of these moduli of the individual layers. Another type of averaging applicable where the thickness of the layers is much larger than the wavelength is the traveltime average (Wyllie, 1956), where the traveltime through a sequence is the sum of the traveltimes through the individual layers. Finally, one can use full-wave equation simulators or, alternatively, relatively simple convolutional computations to obtain the seismic signatures of a formation comprised of subresolution layers whose elastic properties are known. To obtain the effective fluid flow or electrical current properties of a large volume from those of its subvolumes, conceptually similar techniques, generally called reservoir simulations, are used.

Upscaling and averaging are two different concepts. Mathematical averaging can be conducted in many ways, including arithmetic and harmonic averages. None of the averaging methods, unless supported by the underlying physics, does not necessarily produce the physical properties of a composite made of individual elements. Here, we numerically simulate the actual physical properties of a composite by using either the rigorous elastic bounds or Darcy’s flow simulation. Hence, the results thus obtained are relevant to physical upscaling rather than mathematical averaging.

A wealth of work has been dedicated to estimating the effective properties of composites. For example, Neman-Nasser and Hori (1993) provide an exhaustive treatise for the overall elastic properties of composites based on micromechanics. Torquato (2000) presents an extensive mathematical apparatus for deriving the macroscopic properties from microstructure in random heterogeneous materials. Kachanov and Sevostianov (2005) focus on microstructures that can be characterized as continuous matrices containing inclusions of different properties. They discuss how individual inhomogeneities contribute to the effective properties. The possibility of cross-property relations is addressed in this work based on the microstructural parameter similarity. Grechka and Kachanov (2006) address the effective elasticity of fractured rocks by comparing the predictions of two theories for fractured formations and addressing the differences between them.

The question we pose here is different. Instead of obtaining a property (volumetric, elastic, or transport) of a large volume from the same property of its subvolumes, we wish to know how to obtain...
a certain property of the large volume from another property of the same volume, without knowing the details of spatial distributions of these properties. We hypothesize that such a transform between two or more properties of the large volume (often called the effective properties) can be derived from the transform that relates these properties of the subvolumes, no matter how these subvolumes are arranged in space or what the exact property values are ascribed to each subvolume. In other words, we wish to derive a rock physics transform between the effective properties from the transform established for the constituents.

Moreover, instead of concentrating on the microstructure of the material, we accept the properties of the elements of a composite at their face value — as measured. Arguably, this approach can be termed as “meso-to-macro” rather than “micro-to-macro.” However, the terminology is not as important as the method; we simulate physical experiments on a composite and arrive at results that suggest that under certain assumptions, if a pair of the physical properties for all the elements of a composite form a trend, the same trend may be valid for the composite.

There are two important assumptions used in the following discussion and examples:

- All the elements of a composite have to approximately obey the same cross-property trend. In other words, the measured material properties controlling a particular cross-property relation have the same statistics.
- The elements of the composite are spatially uncorrelated. This means that the composite is approximately isotropic (rather than, e.g., layered), at least in cases where it is constructed from a large number of the elements.

A typical example is transforming the large-scale elastic velocity or impedance into the large-scale porosity if a transform is established from controlled experiments (well, physical, or digital laboratory), where the velocity and porosity are measured simultaneously at a much smaller scale and on several of the subsamples sufficient to produce a transform. Traditionally, such exportability of rock physics transforms is taken for granted. Whether or not it can be will become clear from the following discussion.

**IMPEDEANCE-POROSITY AND IMPEDEANCE-POISSON’S RATIO**

Consider a subset of Han’s (1986) porosity and velocity measurements obtained on room-dry sandstone samples at 30 MPa confining pressure and for the clay content varying between 3 and 11%. In Figure 1a, we display the P-wave impedance $I_p$ versus porosity $\phi$ and, in Figure 1b, $I_p$ versus Poisson’s ratio $\nu$ for this data set of 28 samples. In the same figure, we plot the wet-rock elastic properties computed from the room-dry data by Gassmann’s (1951) fluid substitution, where the assumed bulk modulus of water is 2.65 GPa and its density is 1.00 g/cc.

Let us next assume that a number $m$ of samples from this data set are combined to create a hypothetical volume. Also, for simplicity, assume that the volume fraction of each subsample (or component) is the same and equals $m^{-1}$. Hence, we can compute the porosity of this composite volume as the arithmetic mean of the $m$ individual subsample porosities and its bulk density as the arithmetic mean of the individual $m$ bulk densities.

To bracket the effective elastic properties of the composite volume, let us apply the Hashin-Shtrikman (1963) upper and lower bounds for the dry-rock bulk and shear moduli of the $m$ elastic components. Notice that if we apply these bounds to all 28 samples of the data set under examination, the upper Hashin-Shtrikman (1963) bounds (HS) for the bulk and shear modulus are 14.81 and 13.73 GPa, respectively, while the lower bounds are 14.37 and 13.21 GPa, respectively. This observed proximity of the upper and lower bounds for both elastic moduli means that any of them, or their means, can be used for the effective elastic moduli of the composite.

Our first trial is to randomly draw $m = 8$ samples from the entire 28-sample set, assume that these eight samples form an elastic composite (Figure 2), and then compute the porosity, density, and the

![Figure 1](#)
HS elastic bounds for this randomly created composite. The bounds for the impedance and Poisson’s ratio of this volume are then computed from its density $\rho$ and the bulk and shear moduli bounds as

$$M_\pm = K_\pm + \frac{4}{3} G_\pm, \quad I_{P\pm} = \sqrt{\rho M_\pm},$$

$$\nu_\pm = \frac{1}{2} \frac{M_\pm}{G_\pm} - \frac{3}{2} \frac{M_\pm}{G_\pm} - 1,$$

(1)

where $M$ is the compressional modulus, $K$ and $G$ are the bulk and shear moduli, and the $\pm$ subscript refers to the upper or lower Hashin-Shtrikman bound, respectively. This trial is repeated about 100 times with each random realization independent of the others. The results of each trial are plotted independently of each other in the impedance versus porosity and impedance versus Poisson’s ratio graphs. This method is statistically homogeneous with no additional disorder introduced.

The resulting dry-rock impedance-porosity and impedance-Poisson’s ratio plots in Figure 3 show that the $(I_P, \phi)$ and $(I_P, \nu)$ pairs computed for each realization of the elastic composite fall upon the trend formed by the same property pairs of the individual samples. The upper and lower HS bounds computed for each realization are so close to each other that they can hardly be separated in Figure 3.

Hence, if the named properties of the components form a trend, the same properties of a larger volume comprised of these components can fall on the same trend. The results for the wet-rock properties displayed in Figure 4 support this conclusion.

In Figure 5, we explore the effect of the number of the elastic components on the trends formed by the randomly constructed composite volumes for $m = 4$, 16, and 200. As we increase the number of the components, the computed property pairs of the composite tend to center around the mean porosity of the components (0.19), the mean impedance (9.23 km/s/g/cc), and Poisson’s ratio (0.21). Still, all the properties thus upscaled comply with the same rock physics trends.

To explore the effect of the proposed trend upscaling for data where the trends are not as tight as shown in Figures 3 and 4, consider the entire data set by Han (1986), where the clay content now varies between zero and 50%. The total number of the physical samples in the data set is now 61. Once again, we use the dry-rock measurements at 30 MPa confining pressure and compute the wet-rock properties by Gassmann’s (1951) fluid substitution. The tight trends between $\phi$ and $I_P$ observed in Figure 4 do not hold for the entire data set, as a result of the effect of clay on the elastic properties (Figure 6).

We repeat the same upscaling procedure by randomly drawing eight data points from the entire 61-sample data set and compute

![Figure 2. Cartoon illustrating random selection of eight samples from the 28-sample data set (a), with the selected samples marked with crosses) and combining these eight samples into a composite volume (b).](image)

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![Figure 3. The dry-rock impedance versus porosity (a) and versus Poisson’s ratio (b) from Figure 1 (light circles). Superimposed upon these data are the upper and lower HS bounds for the impedance versus mean porosity (a) and impedance versus Poisson’s ratio (b) for multiple random realizations of the 8C composite (light squares).](image)

Figure 3. The dry-rock impedance versus porosity (a) and versus Poisson’s ratio (b) from Figure 1 (light circles). Superimposed upon these data are the upper and lower HS bounds for the impedance versus mean porosity (a) and impedance versus Poisson’s ratio (b) for multiple random realizations of the 8C composite (light squares).
the porosity of this eight-sample composite and its elastic bounds as
described for the previous example. These upscaled data are super-
imposed upon the original data set in Figure 7. The data pairs thus
computed fall into the domain occupied by the original data set
without forming a tight trend. This means that if a definite trend
(transform) is absent for the fine-scale data, we should not expect
a trend to appear for the upscaled properties.

A useful property of the data set under examination is that if the
impedance is plotted versus a linear combination of porosity and
clay content (C), such as $\phi + 0.3C$, a trend appears (Figure 8a).
This effect was first noticed by Gal et al. (1999) and attributed
to the fact that some of the clay in these sandstone samples is
not load-bearing and, hence, only acts to reduce the porosity with-
out strongly affecting the elastic properties. This is why, when the
impedance is plotted versus the volume fraction occupied by pores
and clay, it forms a relatively tight trend. Of course, the coefficient
0.3 in front of C is not universal and may change depending on the
data set and the role of clay in the rock fabric. We observe that if the
effective impedance of a randomly constructed elastic composite is
plotted versus its mean porosity plus 0.3 times its mean clay con-
ten, the upscaled properties comply with the trend formed by the
original samples (Figure 8b).

In our next and final example of upscaling, elastic property trans-
forms are based on a rock physics model rather than a data set.
Specifically, we select the soft-sand model (Mavko et al., 2009)
appropriate for unconsolidated clastic formations and assume that
the porosity of a sample can vary between zero and 0.4 whereas its
clay content can vary between zero and 30%. The corresponding
model curves for wet-rock are displayed in Figure 9.

As before, we randomly select eight subsamples within the
specified porosity and clay content ranges and use them as the
equal-fraction (1/8) components of a composite whose elastic

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**Figure 4.** Same as Figure 3 but for wet-rock properties computed from the dry-rock properties as explained in the text.

**Figure 5.** Same as Figure 4 but for random realization with four (black circles), 16 (white circles), and 200 (black white-rimmed squares). The crossplots for the individual components are not shown. The numbers in the plots indicate the number of components used in upscaling.
properties are, once again, bracketed by the upper and lower Hashin-Shtrikman bounds. For each random realization, both bounds for the impedance are plotted versus the composite’s mean porosity (Figure 9a) and versus the bounds for Poisson’s ratio (Figure 9b).

These lower Hashin-Shtrikman computed data pairs fall close to the model trends, hence illustrating once again that a rock physics transform is stable versus the scale of measurement. This statement, of course, is only valid if all the components of a large volume comply with given rock physics transform.

However, in this case where the porosity span is very wide (from zero to 0.40), we also observe that the impedance of the composite volumes exceeds the original elemental impedance versus porosity model curve for zero clay content (Figure 9a). A likely reason for this deviation is the nonlinearity of the underlying impedance-porosity trend. Below, we will further explore the effects of such nonlinearity on the behavior of the upscaled trends.

**PERMEABILITY-POROSITY**

Consider a hypothetical porous and permeable volume (reservoir) that is comprised of porous and permeable elements. Assume (only for simplicity) that the reservoir is a cube formed by \( n^3 \) subcubes and the volumes of each of these subcubes are the same (Figure 10).

Let us also assume that the porosity and permeability of each subcube is known and define the porosity of the reservoir as the arithmetic average of the porosities of the components.

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![Figure 6](image.png)

**Figure 6.** Han’s (1986) data for the samples with the clay content between zero and 50%. (a) Impedance versus porosity. (b) Impedance versus Poisson’s ratio. The symbols are color-coded by the clay content. The wet-rock data used in these plots were computed from the original dry-rock data via Gassmann’s (1951) fluid substitution.

![Figure 7](image.png)

**Figure 7.** Same as Figure 6, but with upscaled properties (light squares) plotted upon the original data. Each composite volume consists of eight subsamples.
Figure 8. Wet-rock impedance versus porosity plus 0.3 times the clay content color-coded by the clay content (a) with the upscaled elastic properties of multiple realizations of the eight-element elastic composite superimposed upon the original data (light squares, b).

Figure 9. The soft-sand model. Wet-rock impedance versus porosity (a) and versus Poisson’s ratio (b). The two curves are for zero and 30% clay content, as marked in the plots. The open squares are for the upper Hashin-Shtrikman bound of 300 realization of an eight-element composite whereas black circles are for lower Hashin-Shtrikman bound.

Figure 10. A reservoir formed by $5 \times 5 \times 5$ porous and permeable subcubes of equal size. The porosity cube (a) has the mean porosity 0.2, minimum porosity 0.1 and maximum porosity 0.3. The permeability cube (b) has the mean permeability 297 mD, minimum permeability 14 mD, and maximum permeability 931 mD. The effective permeability of this reservoir in the vertical direction is 181 mD. In this specific example, the permeability is computed from porosity by using the Kozeny-Carman equation (equation 3 as explained later in the text).
The effective absolute permeability of this reservoir is obtained by using finite differences to numerically solve the single-phase diffusion equation for fluid pressure $P$ in the 3D rectangular coordinate system $(x,y,z)$ with spatially varying absolute permeability $k(x,y,z)$

$$\nabla [k(x,y,z) \nabla P(x,y,z)] = 0,$$

where $\nabla$ is the 3D gradient operator. In this implementation, $k(x,y,z) = k(l,m,n)$, where $(l,m,n)$ are the indices of the subcube on the 3D grid. For fixed $l$, $m$, and $n$, $k(l,m,n)$ is assumed to be uniform and isotropic, meaning that each subcube is characterized by a single porosity and permeability numbers. Of course, the effective permeability of the composite may be anisotropic even if the distribution of the subcubes does not have any spatial structure. The fluid flow in a composite was simulated in one direction with the no-flow boundary conditions on the sides.

Next, we select a laboratory porosity-permeability data set and emulate the workflow we used in upscaling the elastic property trends. We randomly draw subsamples from the data set, combine them into a cube-shaped reservoir, compute its mean porosity and effective permeability, and plot the latter versus the former. The two laboratory data sets we will use are for sand samples from the Troll and Oseberg fields in the North Sea (Blangy, 1992; S. Strandenes, personal communication, 1993). These data are displayed in Figure 11a with the classical Fontainebleau data set (Bourbie and Zinszner, 1985) given for reference. Our computational experiments will be conducted on (Figure 11a) the Oseberg data and (Figure 11b) a subset of the Troll data that forms a tight permeability-porosity trend.

In the first experiment, we randomly select $27 = 3^3$ samples separately from each data set, use them to construct a cube-shaped reservoir, compute its mean porosity and effective permeability for each reservoir in the vertical direction, and plot the latter versus the former. We repeat these random realizations and compute the permeability by using a transform valid for selected subsamples, measured in the physical or digital laboratory. Needless to say, the main assumption is that all the elements of the reservoir obey this transform.

This rule becomes invalid if the background data set does not form a trend. Indeed, if we duplicate our computational experiment for the full Troll data set, the effective permeability and porosity pairs center in the middle of the data cloud and do not form a clear transform (Figure 13).

Finally, as we did in the elasticity section, let us show now that not only data trends but also a model-based transforms persist as the scale of investigation increases. Specifically, let us use the Kozeny-Carman model that relates the permeability $k$ to porosity $\phi$, grain size $d$, tortuosity $\tau$, and the percolation porosity $\phi_p$

$$k = 10^9 \frac{d^2}{2} \frac{(\phi - \phi_p)^3}{(1 - \phi + \phi_p)^2 \tau^2},$$

where the grain size is in mm and permeability is in mD (Mavko et al., 2009).

Specifically, we select $d = 0.10$ mm, $\tau = 2.50$, and $\phi_p = 0.02$. We also constrain the porosity range between 0.10 and 0.30 and construct a $3 \times 3 \times 3$ and a $5 \times 5 \times 5$ reservoir based on the porosity values randomly selected within this range. Then, the permeability of each element is computed from equation 3 while the vertical effective permeability of the reservoir thus produced is computed using the Darcy single-phase flow simulator.

Figure 11. (a) Decimal logarithm of permeability versus porosity for Fontainebleau sandstone and Troll and Oseberg sands (e.g., number 0 on the permeability axis stands for 1 mD while number 3 stands for 1000 mD). (b) The same but only for selected Troll data and Oseberg data.
The results shown in Figure 14 indicate that the Kozeny-Carman permeability-porosity transform holds for a composite reservoir if its elements obey the same transform.

**CONDUCTIVITY-POROSITY**

The electrical conductivity of a composite where the porosity and conductivity of each element are known can be computed by solving the same diffusion equation as for Darcy’s single-phase flow where pressure $P$ is replaced with the electrical potential $U$.

\[
\nabla \left[ k(x, y, z) \nabla U(x, y, z) \right] = 0. \quad (4)
\]

This means that no additional computations are needed to prove that if the conductivity (or formation factor) of the elements of a composite form a trend versus their porosity, the effective conductivity of the composite plotted versus its mean porosity can fall on the same trend. Of course, this conclusion is only valid for simple cases of electrical flow through the electrolyte-filled pore space, excluding such effects as surface conductivity, clays, and conductive minerals.

**LINEARITY CONDITION FOR PRESERVING THE TRENDS**

Most of the trends discussed here are approximately linear. Even if they are nonlinear for the relation between the impedance, porosity, and clay content (Figure 6) but can be made linear by relating the impedance to a combination of porosity and clay content (Figure 8), the upscaled trends follow the elemental trend thus...

Figure 12. Effective permeability versus mean porosity for multiple realizations computed as explained in the text (black hexahedrons) superimposed onto the original physical data (gray circles for Troll and gray squares for Oseberg). A $3 \times 3 \times 3$ cube-shaped reservoir (a) and a $5 \times 5 \times 5$ cube-shaped reservoir (b).

Figure 13. Same as Figure 12, but for the full Troll data set. A $3 \times 3 \times 3$ cube-shaped reservoir (a) and a $5 \times 5 \times 5$ cube-shaped reservoir (b).
obtained. However, for a concave trend between the impedance and porosity generated by the soft-sand model (Figure 9) and for the entire porosity range (zero to 0.40), the upscaled trends can exceed the elemental underlying trend.

Let us further explore the pure effect of strong nonlinearity in an elemental trend on the upscaled trend. For this purpose, we revisit the soft-sand model that produces a concave trend between the impedance and porosity (Figure 15). The input parameters in this example are the same as used for the curves displayed in Figure 9 except that we assume the rock pure quartz with zero clay content. We start by drawing only four elements in each random realization to construct an elastic composite and by separately examining three porosity intervals, one between zero and 0.05, another between 0.30 and 0.40, and the third between zero and 0.40 (Figure 15a). In addition, we construct the elastic composite by using 125 elements (Figure 15b). In the first two intervals, where the underlying model curve is approximately linear, the upscaled trends accurately trace the model curve. However, in the third case where the entire porosity interval is used, while the lower Hashin-Shtrikman bounds follow the model curve, the upper Hashin-Shtrikman bounds for the impedance lie above this curve.

This example indicates that our conclusions are only strictly accurate where the elemental trend is close to linear. Still, even in this case, if we select to compute the upscaled trend as, e.g., the arith-

Figure 14. Permeability upscaling conducted on individual elements whose permeability is related to porosity by equation 3 (the Kozeny-Carman equation). The squares (same in a and in b) show the porosity-permeability domain covered in this test. The curves (same in a and in b) are according to equation 3 for the entire porosity range. The symbols are the effective permeability computed using the Darcy simulator versus the mean porosity of the reservoir for multiple random realizations for a $3 \times 3 \times 3$ reservoir (a) and $5 \times 5 \times 5$ reservoir (b).

Figure 15. Impedance versus porosity for the soft-sand model. The impedance upscaling was conducted for three separate porosity intervals, one from zero to 0.05, another from 0.30 to 0.40, and the third from zero to 0.40 for randomly drawn four- (a) and 16-element (b) elastic composite. We used several hundred random realizations. In both plots, the black curve is the original soft-sand model, black symbols are for the lower Hashin-Shtrikman bound, and open larger symbols are for the upper Hashin-Shtrikman bound. The double arrows indicate the span of the porosity intervals used in these computations.
metric (or harmonic average) between the two bounds, the results will be fairly close to the model line, thus preserving the elemental trend. Moreover, in most practical cases, the porosity range is smaller than 40% porosity units and, hence, the preservation of the elemental trends for a composite can still be valid.

NONSTATIONARY ELEMENTAL TRENDS

We choose the acoustic impedance versus porosity example to explore how the nonstationarity of the elemental trends can affect the trends obtained for a composite constructed of such elements. Specifically, we use two underlying elemental trends, one for the soft-sand model and the other for the stiff-sand model (both models are described in Mavko et al., 2009). The porosity range used in this example is between 0.10 and 0.30. The rock is wet and pure quartz.

The difference between this and previous examples is that here we randomly draw the elements of the elastic composite from the lower or the upper model curve. As before, we use several hundred random realizations for the composite.

The results (Figure 16) indicate that the Hashin-Shtrikman bounds computed for the composite thus constructed fill the entire space between the elemental model curves in the impedance-porosity plane in the case where the composite is comprised of only 4 elements. If the number of the elements is increased from 4 to 125, the cluster of the impedance-porosity pairs for the composite is centered around the average porosity 0.20 and in the middle between the two elemental curves. A conclusion from this example is that if the statistical stationarity of the elemental trend is violated, the upscaled trend may not follow either of the trends formed by the elements.

CONCLUSION

Computational experiments presented here suggest that rock physics transforms can be stable versus the scale of measurement, meaning that a transform can be preserved for the process-driven effective properties of a large volume if all of its elements obey the same transform. The computations used here to illustrate this point are internally consistent, yet the underlying assumptions are stringent; it is unlikely to have controlled physical measurements for each element of a large volume. As it often happens in geophysics, we need to supplement our computational construction with a practical assumption, namely that all elements obey the transform established in controlled experiment on an acceptable (representative) number of subsamples. One criterion for the data set being representative is that it covers a range of porosity possibly encountered in the reservoir.

Another practical approach often used in geophysics is to find a data set from a formation that is geologically similar (analogous) to the reservoir under examination. Such similarity can be established by, e.g., comparing log data from an exploration well to the published (or proprietary) laboratory data. If we assume next that all the elements of the reservoir block obey the laboratory transform, the upsampling rules discussed here can possibly be used in the field. Additional requirements are that the relations between the rock properties of interest are close to linear and a reservoir can be considered a composite of randomly and independently drawn blocks.

The discussion presented here was mainly driven by the emergence of the digital rock physics where the properties are computed on mm-sized (or lesser) tests. The question often asked is how is it possible (if possible at all) to infer rock properties from such minuscule volumes? The answer given here is that the rock properties of a larger body may differ from those of a microscopic sample but the transforms established on several microscopic samples can hold for a larger volume. By extension, the same conclusion is valid if we wish to characterize a large geo-body by measuring several cm-sized samples in the physical laboratory (although the appropriateness of this approach has rarely been questioned).

The method used here is computational experimentation rather than rigorous analytical proof. Still, we find the examples presented...
compelling enough to suggest that basic rock physics transforms can be scale-independent.

Finally, the intention of this work is to propose a principle and show how it can work for the basic rock properties. It still remains unclear whether this principle holds (and if it does, how) for special core analysis properties, such as relative permeability, irreducible water and residual hydrocarbon saturation, and, especially in the presence of strong heterogeneity of the trends as well as the anisotropy of the measured properties. We hope this work presents an avenue for such future development.

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